**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer: B. 0.2676

We have a normal distribution with =45 and =8.0. Let X be the amount of time it takes to complete the repair on a customer’s car. To finish in one hour you must have x less than equal to 50 so the question is to find Pr(X>50).

Pr(X>50)=1-Pr(X less than equal to 50).

Z=(X-)/=(X-45)/8.0

Thus the question can be answered by using the normal table to find

Pr(X less than equal to 50)=Pr(Z less than equal to (50-45)/8.0)=Pr(Z less than equal to 0.625)=73.4%.

Probability that the service manager will not meet his demand will be=100-73.4=26.6% or 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: We have a normal distribution with =38 and =6. Let X be the number of employees. So according to question

a)Probability of employees greater than age of 44=Pr(X less than 44)

Pr(X less than 44)=1-Pr(X less than equal to 44).

Z=(X-)/=(X-38)/6

Thus the question can be answered by using the normal to find

Pr(X less than equal to 44)=Pr(Z less than equal to(44-38)/6=Pr(Z less than equal to 1)=84.1345%.

Probability that the employee will be greater than age of 44=100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age=Pr(X<44)-0.5=84.1345-0.5=34.1345%

Therefore the statement that “More employees at the processing center are older than 44 than between 38 and 44” is TRUE.

b) Probability of employees less than age of 30=Pr(X<30).

Z=(X-)/=(30-38)/6

Thus the question can be answered by using the normal table to find

Pr(X less than equal to 30)=Pr(Z less than equal to (30-38)/6)=Pr(Z less than equal to - 1.333)=9.12%

So the number of employees with probability 0.912 of them being under age 30=0.0912\*400=36.48(or 36 employees).

Therefore the statement B of the question is also TRUE.

3.If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

As we know that if X~ N(MU1,Sigma1^2) and Y~N(MU2,Sigma2^2) are two independent random variables then X+Y~N(MU1+MU2,Sigma1+2=Sigma2+2), and X-Y~N(MU1-MU2,Sigma1+2+Sigma2+2).

Similarly if Z=aX+bY, where X and Y are as defined above, i.e Z is linear combination of X and Y, then Z~ N(aMU1+bMU2,a^2Sigma1^2+b^2Sigma2^2)

2X1~ N(2 MU,4 Sigma^2) and

X1+X2~ N(MU+MU,Sigma^2+Sigma^2)~ N(2 MU,2Sigma^2)

2X1-(X1+X2)=N(4MU,6 Sigma^2)

4.Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

Answer:

Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

The probability of getting value between a and b should be 0.99.

So the probability of going wrong, or the probability outside the a and b area is 0.001 (i.e. 1-0.99).

The probability towards left from a=-0.005(i.e.0.01/2).

The probability towards right from b=+0.005(i.e.0.01/2).

So since we have the probability of a and b, need to calculate X, the random variable at a and b which has got these probabilities.

By finding the standard normal variable Z(Z value), we can calculate the X values.

Z=(X-MU)/Sigma

For probability 0.005 the Z value is -2.57(from Z table).

Z\*Sigma +MU =X

Z(-0.005)\*20+100=-(-2.57)\*20+100=151.4.

Z(+0.005)\*20+100=-(-2.57)\*20+100=48.6

So, option D is correct.

5.Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
2. Specify the 5th percentile of profit (in Rupees) for the company
3. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

Total profit=profit1+profit2=P

Hence P~ N(12,74)=P-12/square root of 74 .N(0,1)

1. Specifying a Rupee range(centered on the mean) that contains 95% probability for annual profit of the company

$13.41=Rs 603.68

1. Specifying the 5th percentile of profit

P=$10.59~~Rs 476.33

1. The division that has a larger probability of making a loss in a given year is the first division